Radial Basis Function Approximation Methods

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Introduction

- RBFs first studied by Roland Hardy - 1968

$$s(x) = \sum_{j=1}^{N} \lambda_j \phi(\| x - x_j^c \|_2, \epsilon)$$

- Allow for scattered data sites to be easily used in computations
- Used to represent topographical surfaces and other 3-D shapes
  - Facial recognition
  - Ocean floor mapping
  - Medical applications

The most popular RBF that is used in applications today is the Multiquadric (MQ)

$$\phi(r) = \sqrt{1 + \varepsilon^2 r^2} = (1 + \varepsilon^2 r^2)^{1/2}$$

- Properties of the MQ are well-known [2]

A related RBF with properties not as well-known is the Generalized Multiquadric (GMQ)

$$\phi(r) = (1 + \varepsilon^2 r^2)^{\beta} \qquad \beta = \cdots \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2} \cdots$$

- Researchers have recently suggested, but not proven, that the GMQ has desirable properties for β with non-half-integer powers.

GMQ - Suggested Values for β

- Wang and Liu [3]: β = 1.03
- Xaio and McCarthy [4]: β = 1.99
- Kansa [1]: β = 5/2

Which value is best?

Two test functions:

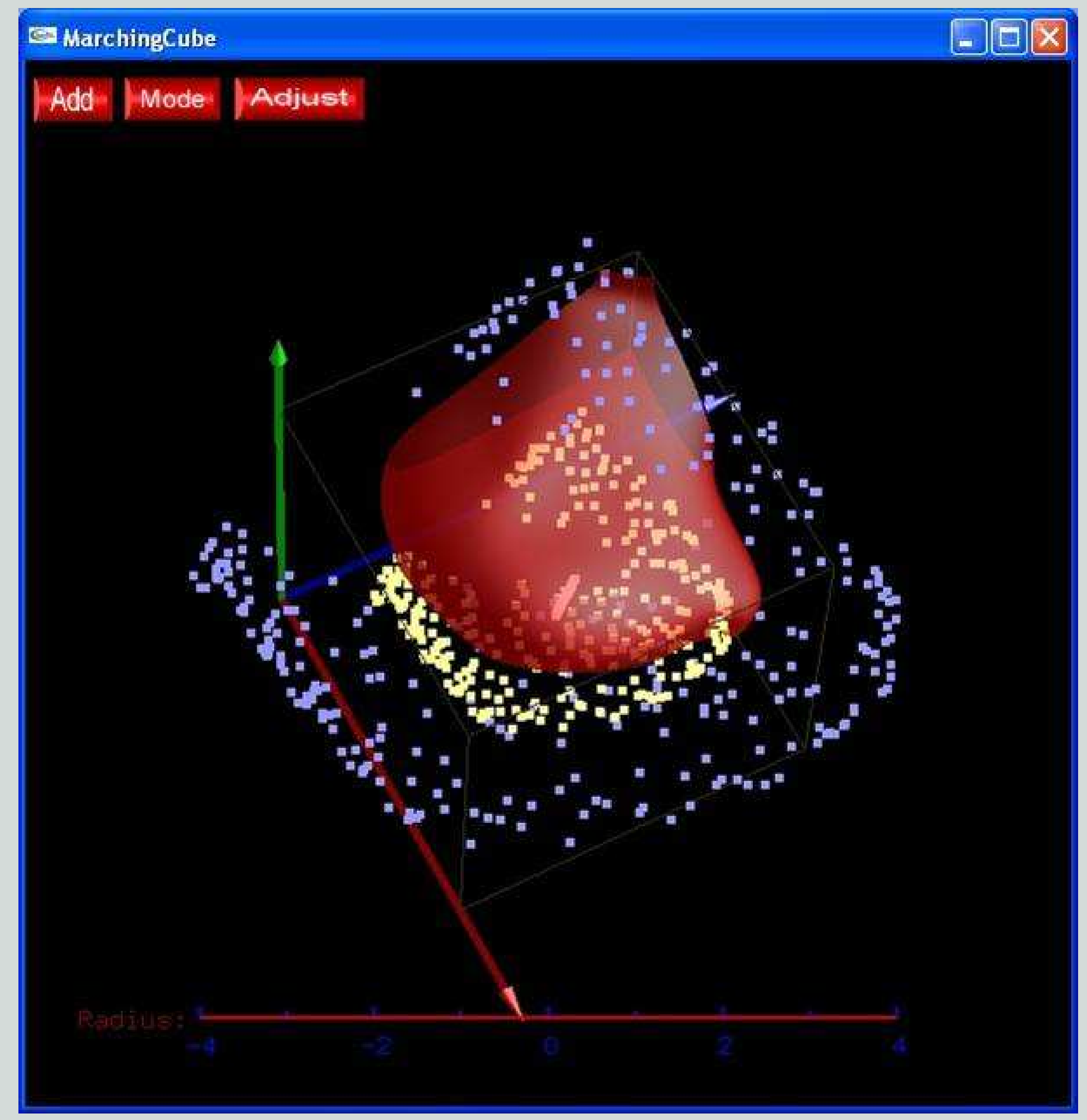
$$f(x) = e^{x^3} + \cos(2x) \qquad (1)$$

$$f(x) = x^4 + 3x^2 - x - 2, \qquad (2)$$

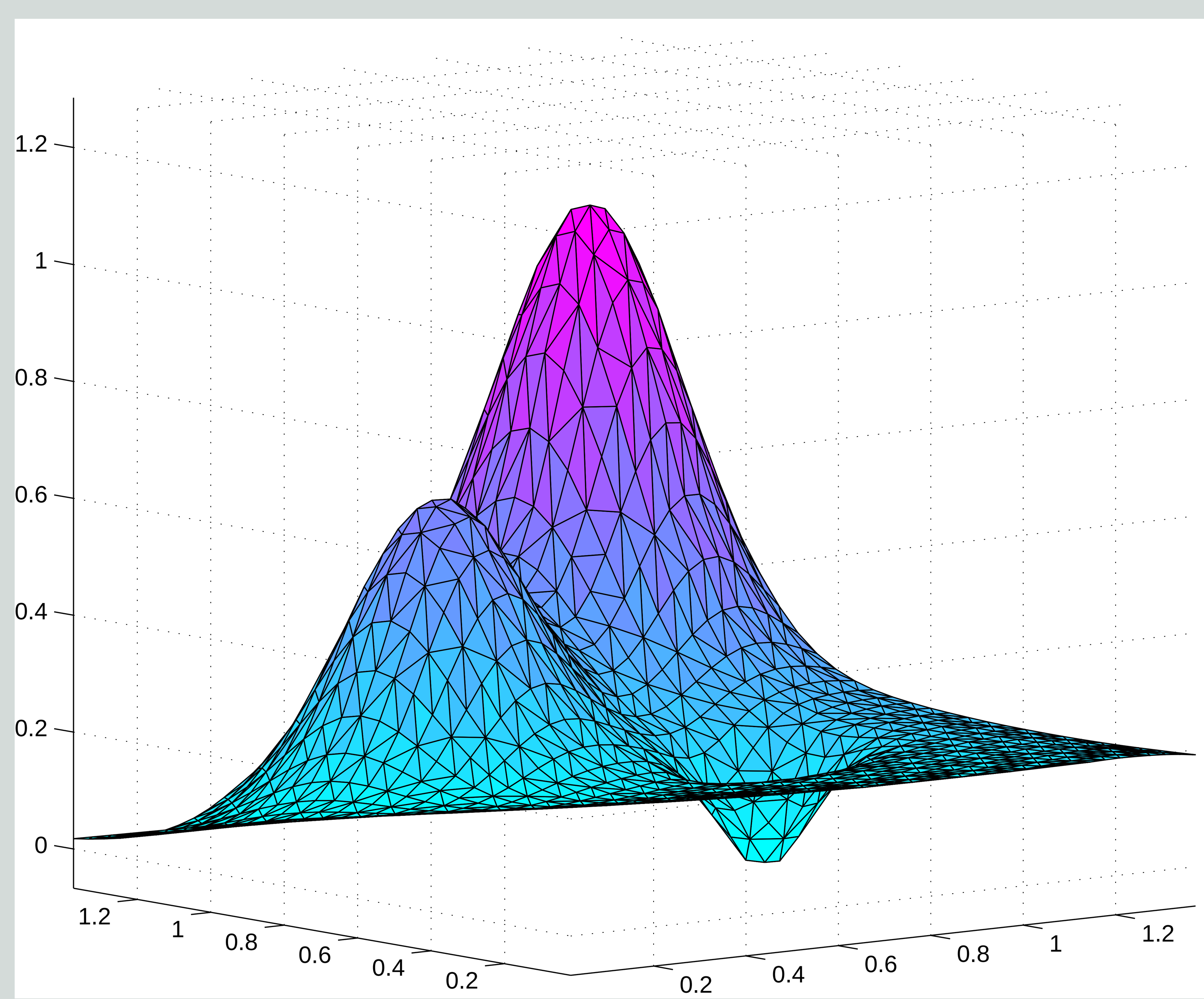
Results:

- No optimal shape parameter was found for the six values of β
- As beta increases the shape parameters increase
- $\beta = 2.5$ provided the best results for the second function, but $\beta = 1.99$ provided the best results for the first function
- β is problem dependent

RBF Reconstruction of Ventricular Surface



Function Generated Using GMQ Interpolation



Extended Precision

RBF methods can be accurately and efficiently evaluated using extended precision floating point arithmetic

- Implemented using computer software instead of hardware
- Takes more computation time
- Provides better accuracy

Comparison of Floating Point Types

type	bits	p	dps	exec time
double	64	53	16	1
double-double	128	106	32	10
quad-double	256	212	64	100

p = bits in decimal section

dps = accurate decimal places

Using the following function

$$f(x) = e^{\sin(\pi x)} \qquad (3)$$

we found that the 256 quad-double appears to be best

Summary

- Optimal values for β are problem dependent
- Extended precision adds accuracy to RBF methods
  - Multi-core architecture of modern computers
  - Domain decomposition approaches

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References

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[4] J. R. Xaio and M. A. McCarthy. A local heaviside weighted meshless method for two-dimensional solids using radial basis functions. *Computational Mechanics*, 31:301–315, 2003.