The Matlab Postprocessing Toolkit (MPT) User Manual

The MPT user manual first lists all Matlab functions associated with each edge detection and postprocessing algorithm. For each algorithm there is also an example script that demonstrates how the Matlab function can be used that implements the algorithm. For each example script, there is a figure that shows the graphical output of the script.

1 Edge Detection
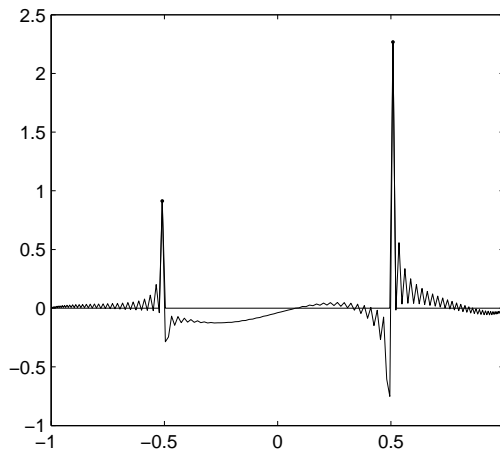

Figure 1: Un-enhanced and enhanced edge detection data produced by edgeDetectChebyshev_example.m using the exponential concentration factor. The following edge detection parameters were used: $Q = 2$, $J = 30$, and $\eta = 2$.

Matlab files: (Note: the inputs and outputs of all the MPT functions are documented in the m-files)

1. edgeDetectChebyshev.m

2. edgeDetectFourier.m

3. edgeDetectChebyshev_example.m - The example applies the edge detection routine to the Chebyshev approximation of the function

$$f(x) = \begin{cases} \cos(\pi x/2) & x < -0.5 \\ x^3 - \sin(3\pi x/2) & -0.5 \leq x \leq 0.5 \\ x^2 + 4x^3 - 5x & x > 0.5 \end{cases} \tag{1}$$

The output is shown in figure 1.

2 Spectral Filter

The Gibbs phenomenon is illustrated in figure 2 for both the Fourier and Chebyshev approximation of the function

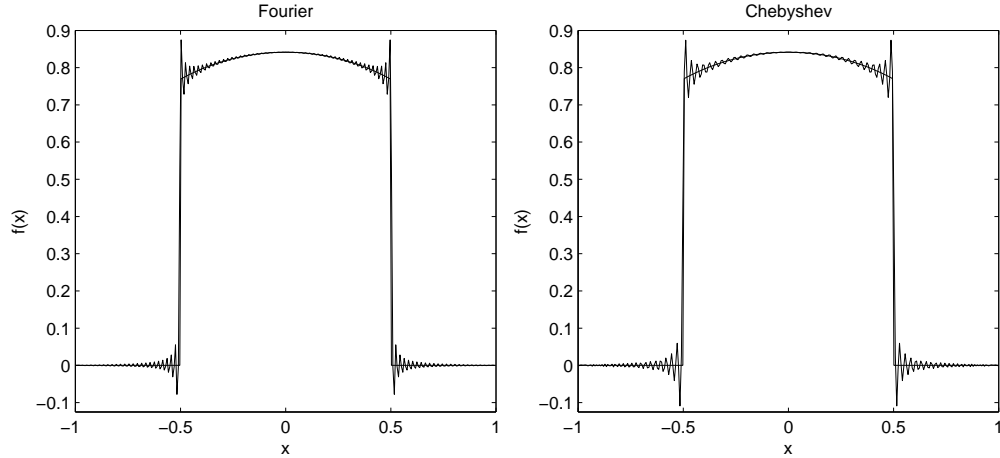$$f(x) = \chi_{[-0.5,0.5]} * \sin[\cos(x)] \tag{2}$$

Figure 2: Spectral approximation of function (2) vs. the exact function. The function is known at $N = 200$ interpolation sites and the interpolant is evaluated at $M = 298$ evenly spaced points. Left: Fourier. Right: Chebyshev.

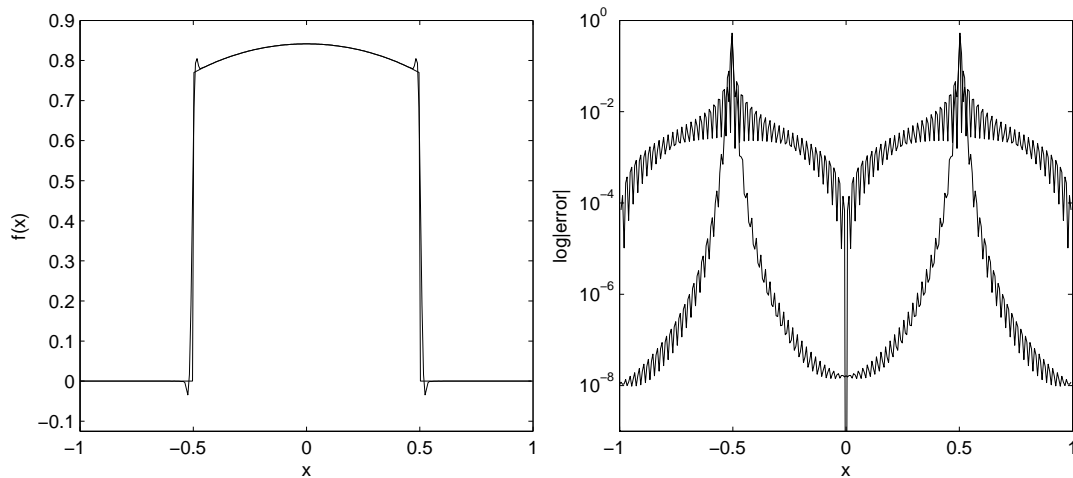that will be used throughout to demonstrate the software.

Figure 3: Output produced by spectralFilter_example.m. Left: Function (2), and the filtered Fourier approximation of the function. Right: Error of the Fourier approximation and the reduced error of the filtered approximation.

Matlab files:

1. spectralFilter.m

2. spectralFilter_example.m - The example calculates the Fourier approximation of the function (2) and the filtered approximation using the fourth-order Vandeven filter. The output is shown in figure 3. The right image shows the increased accuracy of the filtered approximation away from the discontinuity. The oscillatory Fourier approximation is in the left image of figure 2.

3. filterFourier.m

4. filterFourier2d.m

5. filterChebyshev.m

6. filterChebyshev2d.m

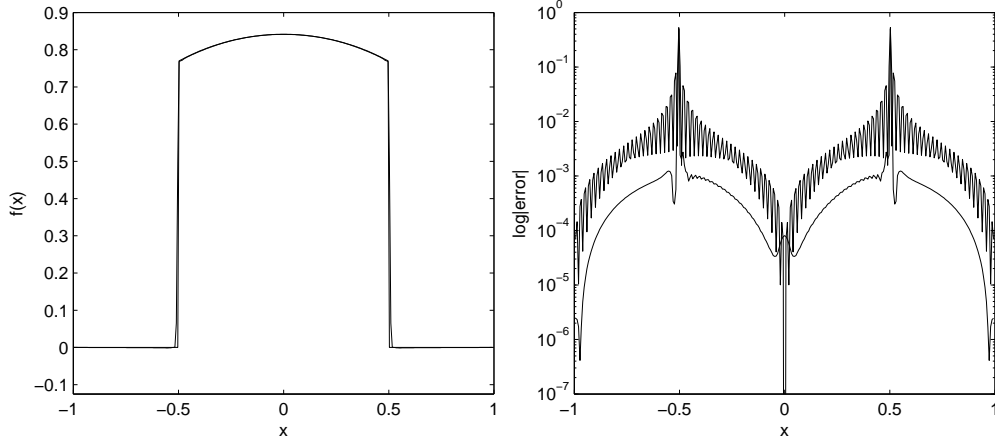# 3   Digital Total Variation Filtering

Figure 4: Output produced by dtvFilter_example.m. Left: Function (2) and the DTV postprocessed Fourier approximation. Right: Error of the Fourier approximation and the reduced error of the DTV postprocessed approximation.

Matlab files:

1. digitalTotalVariationFilter.m - 1d non-periodic

2. digitalTotalVariationFilterPeriodic.m - 1d periodic

3. digitalTotalVariationFilter_2d.m - 2d non-periodic using N_α^4

4. digitalTotalVariationFilter_2d_8.m - 2d non-periodic using N_α^8

5. digitalTotalVariationFilterPeriodic_2d.m - 2d periodic using N_α^4

6. digitalTotalVariationFilterPeriodic_2d_8.m - 2d periodic using N_α^4

7. fourierDTV_example.m - The example postprocesses the Fourier approximation of function (2) using the DTV filter (digitalTotalVariationFilterPeriodic.m) with $\lambda = 10$ and 100 time steps. The output is shown in figure 4. The oscillatory Fourier approximation is in the left image of figure 2.

8. periodic_dtv_example_2d.m - The example postprocesses the Fourier approximation of the function
$$f(x) = \begin{cases} 1 & x^2 + y^2 \leq \frac{1}{4} \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$
using the DTV filter (digitalTotalVariationFilterPeriodic.m) with $\lambda = 10$ and 100 time steps. The output is shown in figure 5. The function is sampled on a 64×64 grid and evaluated on a uniformly spaced 98×98 grid.
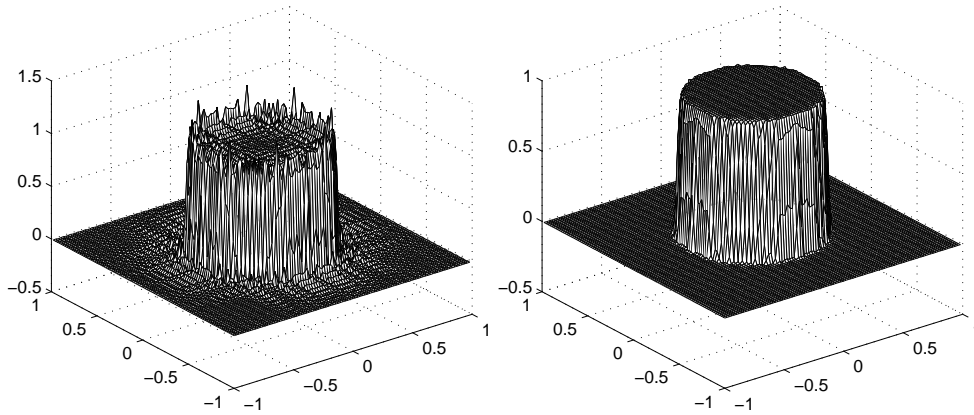
Figure 5: Output produced by periodic_dtv_example_2d.m. Left: Fourier approxima-
tion of function (3). Right: The DTV postprocessed Fourier approximation using
digitalTotalVariationFilterPeriodic_2d.m.

4 Rational Reconstruction

Matlab files:

1. fourierPade.m

2. chebyshevPade.m

3. chebyshevPade_example.m - Figure 6 shows the output of the chebyshevPade_example.m
   which applies chebyshevPade.m with $M = 14$ to postprocess function (2). The
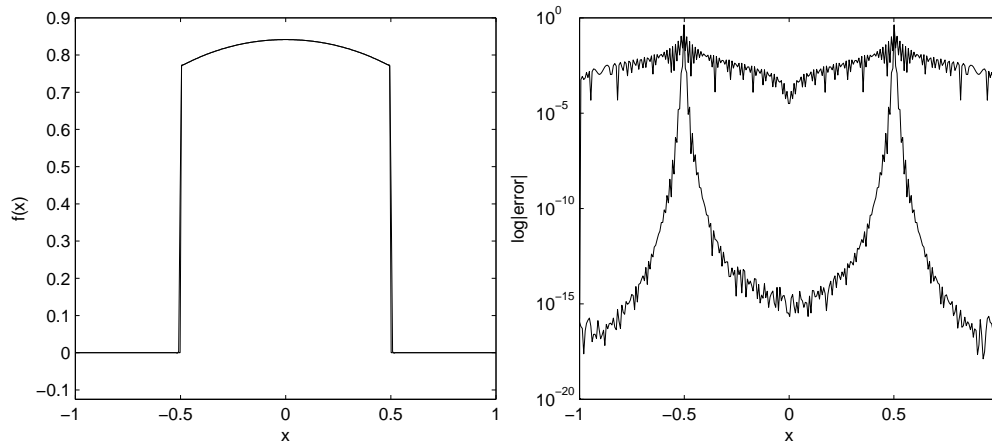   oscillatory Chebyshev approximation is in the right image of figure 2.

Figure 6: Output produced by chebyshevPade_example.m. Left: Function (2), and
the postprocessed approximation of the function. Right: Error of the Chebyshev
approximation and the reduced error of the postprocessed approximation.
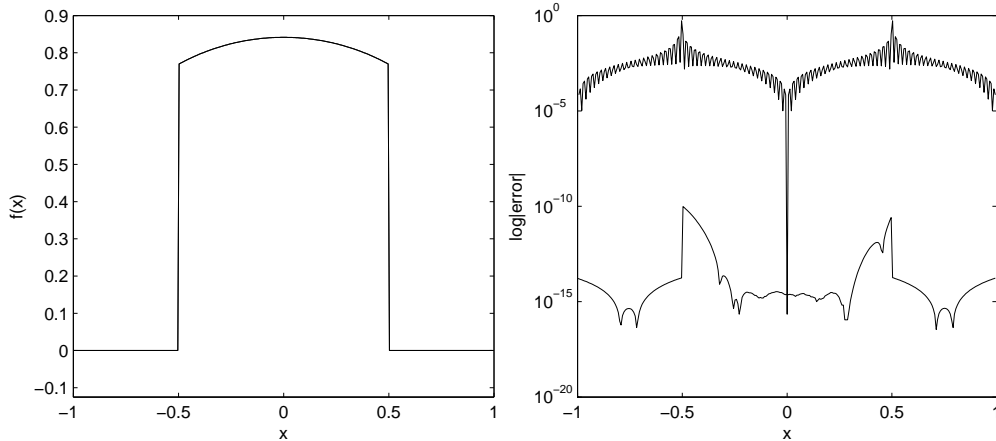
5 Gegenbauer Reprojection

Matlab files:

Figure 7: Output produced by fourierGRP_example.m. Left: Function (2) and the GRP postprocessed Fourier approximation (visually indistinguishable). Right: Error of the Fourier approximation and the reduced error of the postprocessed GRP approximation.

1. grp.m

2. fourierGRP_example.m - see figure 7. The example has three smooth subintervals $\Omega_1 = [-1, -0.5)$, $\Omega_2 = (-0.5, 0.5)$, and $\Omega_3 = (0.5, 1]$. The example used the following GRP parameters: $m_1 = m_2 = m_3 = 13$, $\lambda_1 = 4$, $\lambda_2 = 7$, and $\lambda_3 = 4$. The oscillatory Fourier approximation is in the left image of figure 2.

3. chebyshevSV_eulerDensity_grp.m - Applies the GRP to postprocess the $N = 128$ Chebyshev spectral viscosity approximation of the density profile of Sod's problem [?] for the Euler equations of gas dynamics. The output is in figure 8. The example has five smooth intervals: $\Omega_1 = [-1, -0.4456)$, $\Omega_2 = (-0.4456, -0.0222)$, $\Omega_3 = (-0.0222, 0.3461)$, $\Omega_4 = (0.3461, 0.6606)$, and $\Omega_5 = (0.6606, 1]$. The following GRP parameters were used in the example: $m_1 = 4$, $m_2 = 12$, $m_3 = 4$, $m_4 = 4$, $m_5 = 4$, $\lambda_1 = 5$, $\lambda_2 = 6$, $\lambda_3 = 6$, $\lambda_4 = 4$, and $\lambda_5 = 4$.

6 Freud Reprojection

Matlab files:

1. frp.m

2. freudPolynomials.m

3. fourierFreud_example.m - The script uses the FRP to postprocess the oscillatory Fourier approximation in the left image of figure 2. The results are shown in figure 9. The following FRP parameters were used: $m_1 = 2$, $m_2 = 12$, $m_3 = 2$, $\lambda_1 = 7$, $\lambda_2 = 11$, and $\lambda_3 = 7$.
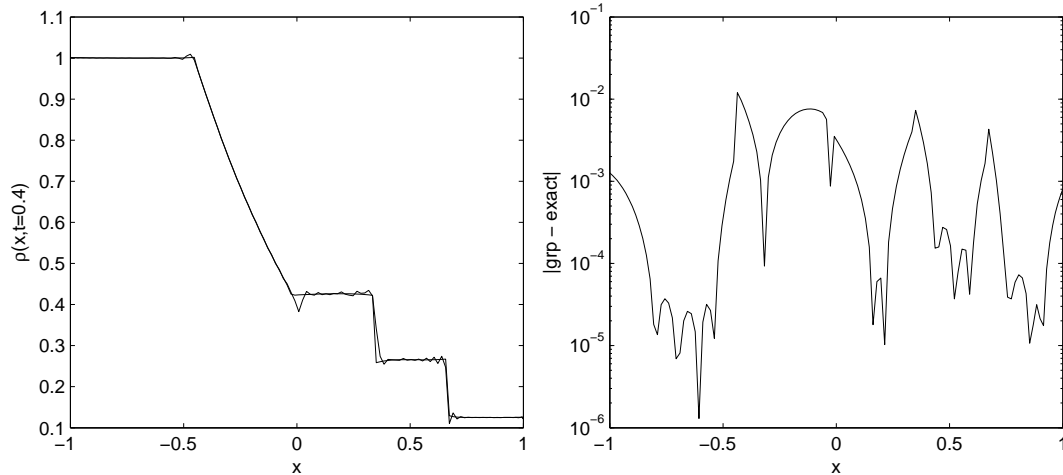
Figure 8: Output of chebyshevSV_eulerDensity_grp.m. Left: Chebyshev spectral viscosity and GRP postprocessed Euler density solution at time t = 0.4. Right: Error of the postprocessed solution.

7   Inverse Reprojection

Matlab files:

1. inverseReprojection.m

2. fourierIPRM_example.m - This script uses the IPRM to postprocess the oscillatory Fourier approximation in the left image of figure 2. The output is shown in figure 10. The following IPRM parameters were used: $m_1 = 2$, $m_2 = 6$, and $m_3 = 2$.
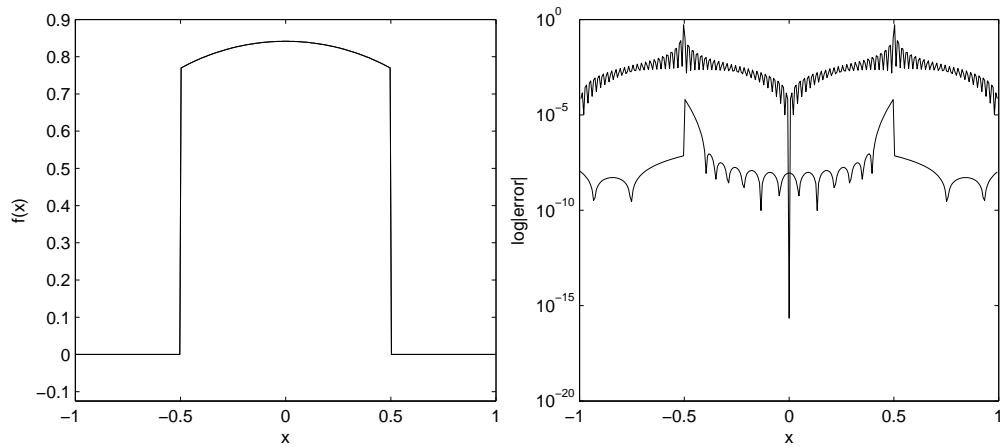
Figure 9: Output produced by fourierFreud_example.m. Left: Function (2) and the FRP postprocessed Fourier approximation (visually indistinguishable). Right: Error of the Fourier approximation and the reduced error of the postprocessed FRP approximation.
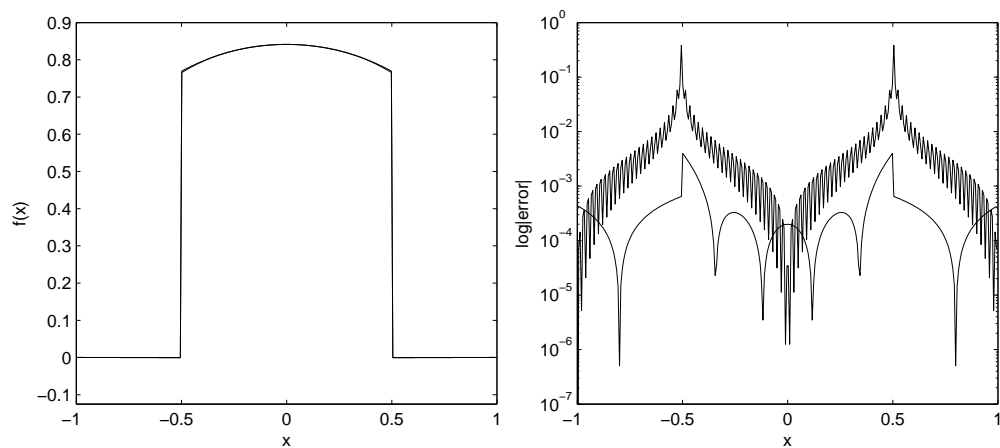

Figure 10: Output produced by fourierIPRM_example.m. Left: Function (2) and the IPRM postprocessed Fourier approximation. Right: Error of the Fourier approximation and the reduced error of the postprocessed IPRM approximation.