



The Matlab Postprocessing Toolbox (MPT)

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Spectral Accuracy/Gibbs

Spectral methods (function interpolation and collocation methods for PDEs) based on global polynomial (e.g. Fourier or Chebyshev polynomials) interpolation yield extremely accurate approximations of smooth functions (spectral or exponential accuracy). However, when functions are only piecewise smooth the well-known *Gibbs phenomenon* appears as an accuracy reduction to first order away from discontinuities and $\mathcal{O}(1)$ oscillations in the neighborhoods of jumps. Several types of postprocessing methods have been developed to lessen the effects of the Gibbs phenomenon or even to restore spectral accuracy. The most powerful of the methods require that the locations of the discontinuities (edges) be precisely known.

Postprocessing Methods

- Spectral Filters
- Digital Total Variation (DTV) Filters
- Gegenbauer Reprojection (GRP)
- Inverse Gegenbauer Reprojection (IGRP)
- Freud Reprojection (FRP)
- Rational (Padé) Reconstruction

A description of the algorithm and references to papers where the algorithms were developed can be found in [1].

Example

As an example we consider the piecewise defined function

$$f(x) = \begin{cases} 1 & -0.4 \leq x \leq -0.2 \\ \frac{15}{4}x + 1 & -0.2 \leq x < 0 \\ -\frac{15}{4}x + 1 & 0 \leq x \leq 0.2 \\ 1 & 0.2 \leq x \leq 0.4 \\ 0 & \text{otherwise} \end{cases}$$

that has a W shaped graph. In the image to the right an oscillatory Chebyshev approximation of the function is shown. Rational reconstruction is used as the postprocessing algorithm. A semilogy plot showing the point-wise error of the Chebyshev approximation and the postprocessed approximation is in the bottom part of the image to the right. In this example rational reconstruction recovers accuracy sufficiently far away from discontinuities and sharply resolves edges.

MPT Overview

The MPT is a collection of Matlab programs that implement state of the art postprocessing and edge detection algorithms for Fourier and Chebyshev spectral approximations of piecewise smooth functions in one and two space dimensions. The MPT functions may be called from a Matlab script or the routines may be accessed from a GUI. The postprocessing functions and accompanying GUI with built-in example functions and PDE solutions provide users the opportunity to benchmark and demonstrate the postprocessing algorithms. Experienced Matlab users will find it easy to modify the GUI to incorporate their own algorithms or example problems. The home of the MPT on the web is www.scottsarra.org/mpt/mpt.html [2].

Postprocessing Method Features

Criteria for comparing postprocessing methods include:

- Do locations of discontinuities need to be located exactly
- The number of parameters to be specified in each region of smoothness
- Is spectral accuracy recovered
- Does an ill-conditioned linear system need to be solved
- Is noise tolerated or are exact spectral coefficients needed
- Computational efficiency

Postprocessing Method Summary

The following table summarizes the basic feature of the postprocessing algorithms:

method	edge detection	parameters	spectral accuracy	large $\kappa(A)$	noise
spectral filter		ρ	\sim		\checkmark
DTV		λ			\checkmark
Padé	\sim	M, N_c		\checkmark	\checkmark
GRP	\checkmark	λ, m_i	\checkmark		\checkmark
FRP	\checkmark		\checkmark		\checkmark
IPRM	\checkmark	m_i	\checkmark	\checkmark	

A \checkmark in the edge detection column indicates the method must know the exact location of the discontinuities while a \sim indicates that the edge location may be incorporated to improve the method. The parameters column lists any user specified parameters. If the method incorporates edge detection the parameters must be specified in each subinterval of smoothness. A \checkmark in the spectral accuracy indicates the method is able to recover spectral accuracy over the entire interval while a \sim indicates that spectral accuracy may be recovered over a portion of the interval sufficiently away from the edge locations. A \checkmark in the large $\kappa(A)$ indicates that a linear system must be solved to implement the method and that the matrix may have a large condition number. Finally, a \checkmark in the noise column indicates that the method can use the noisy spectral coefficients from approximating a discontinuous function and not the exact spectral coefficients of the function or the exact function values.

Summary/Future Directions

The availability of the MPT should foster the idea of reproducible research in the area. The MPT features a Graphical User Interface that allows easy access to the postprocessing algorithms for benchmarking and educational purposes. The MPT will be updated to reflect further developments.

References

- [1] S. A. Sarra. The matlab postprocessing toolkit. *Submitted to ACM Transactions on Mathematical Software*, 2008. www.scottsarra.org/mpt/mpt.html.
- [2] Matlab Postprocessing Toolbox Website. <http://www.scottsarra.org/mpt/mpt.html>.

Graphical User Interface (GUI)

